Application des éléments discrets et de la reconstruction par covariants à la modélisation de l'endommagement anisotrope

Réunion thématique du GDR-GDM, ENS Paris-Saclay, 23/11/2022.

Flavien Loiseau – flavien.loiseau@ens-paris-saclay.fr
Encadré par: Rodrigue Desmorat, Cécile Oliver-Leblond

Université Paris-Saclay, CentraleSupélec, ENS Paris-Saclay, CNRS, LMPS - Laboratoire de Mécanique Paris-Saclay, 91190, Gif-sur-Yvette, France.

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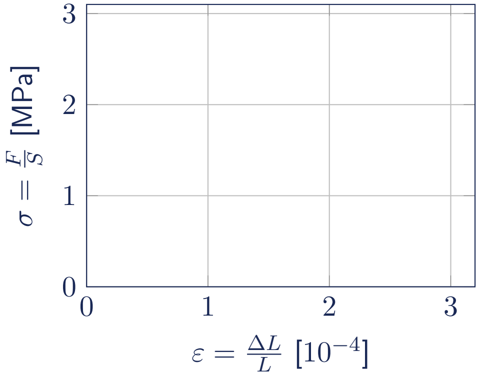
Context

Quasi-brittle materials: Observations

An exemple of structure

A tensile test on concrete (Terrien, 1980)



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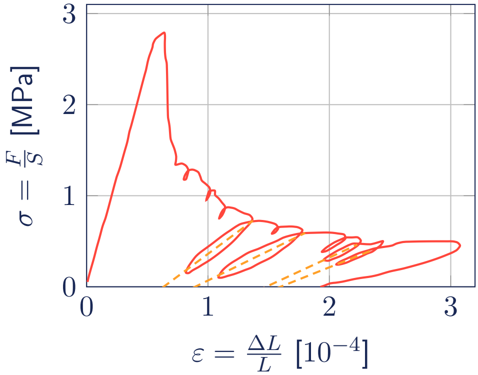
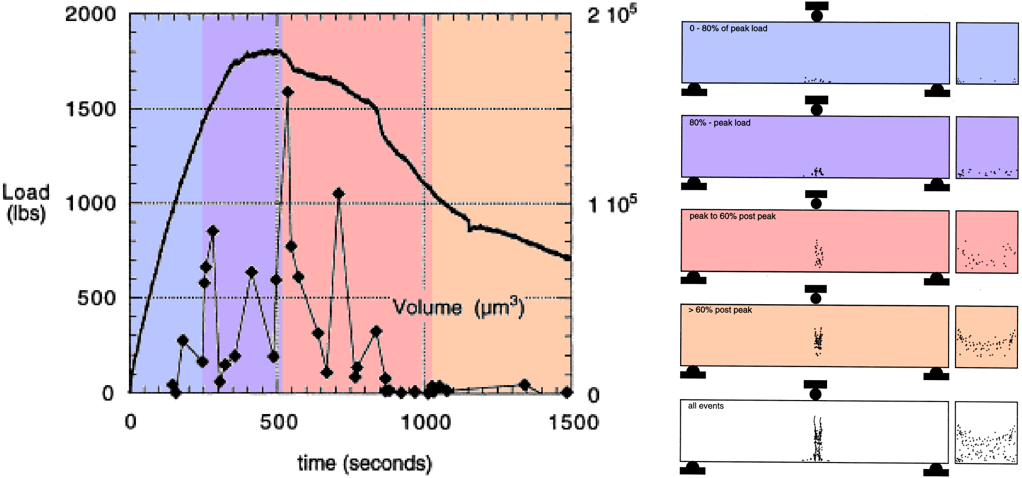


Quasi-brittle materials: Damaging process (Landis, 1999)



Objectives

Objective of the project

Formulating an anisotropic damage model for quasi-brittle material

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Structure of a damage model

$$\mathcal{V} = \{\boldsymbol{\varepsilon}, \mathbf{D}, ...\} \qquad \text{(Set of variables)}$$

$$\rho\psi = \frac{1}{2}\boldsymbol{\varepsilon} : \mathbf{E}(\mathbf{D}) : \boldsymbol{\varepsilon} \qquad \text{(State potential)}$$

$$\frac{\partial \mathbf{D}}{\partial t} = ... \qquad \text{(Damage evolution)}$$

Notations

- $\mathbf{D}$ damage variable
- $\mathbf{E}(\mathbf{D})$ effective elasticity tensor

Constraints

- $\mathbf{E}(\mathbf{D})$ is positive definite
- Positive dissipation

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- $\mathbf{E}(\mathbf{D})$ effective elasticity tensor

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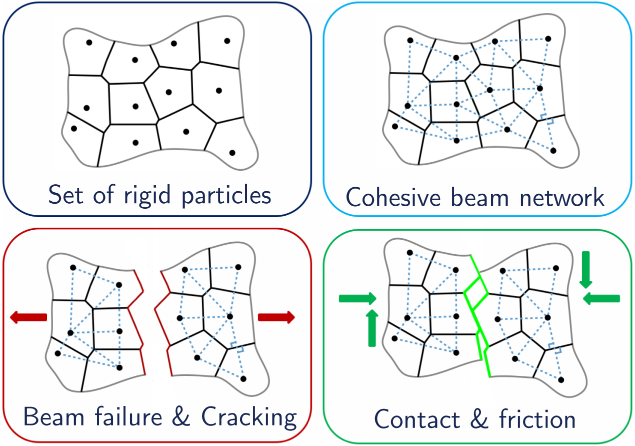
Objectives of the presentation

1. Gather data on the behavior of quasi-brittle material
2. Quantify the micro-cracking
3. Model the impact of micro-cracking on the relation between ε and $\boldsymbol{\sigma}$

Virtual testing

Discrete model – Beam-particle model (Vassaux et al., 2016)



Related prior work: (D'Addetta et al., 2002), (Delaplace, 2008), ...

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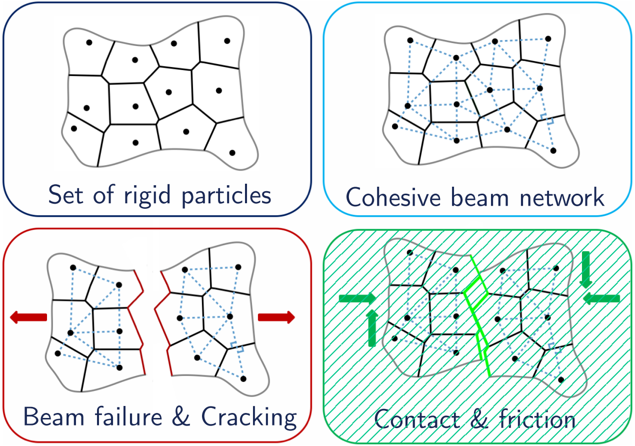

Remark

Contact/friction disabled.

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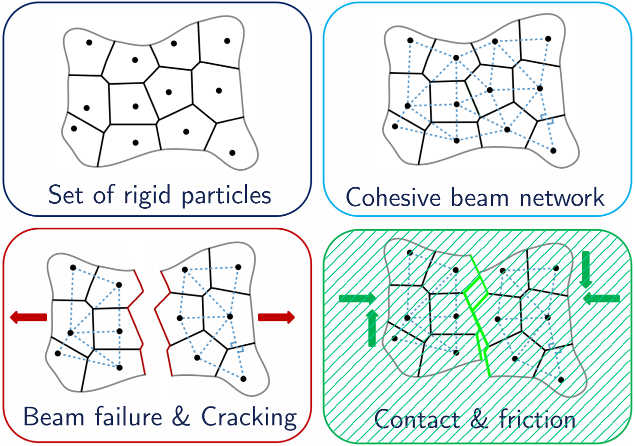

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Info

- ✓ Accurate representation of fracture process (Oliver-Leblond, 2019)
- ✓ Reproductibilty
- ✗ Complex to identify

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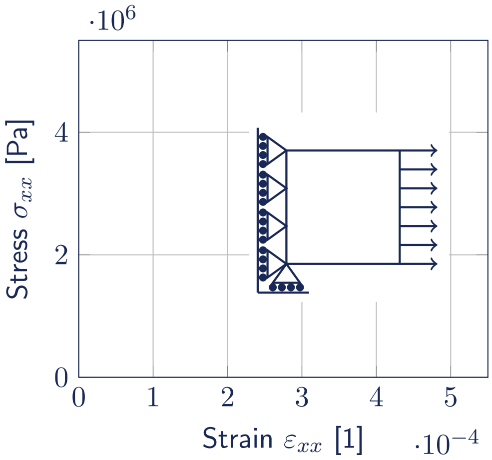
Illustration for a tensile test



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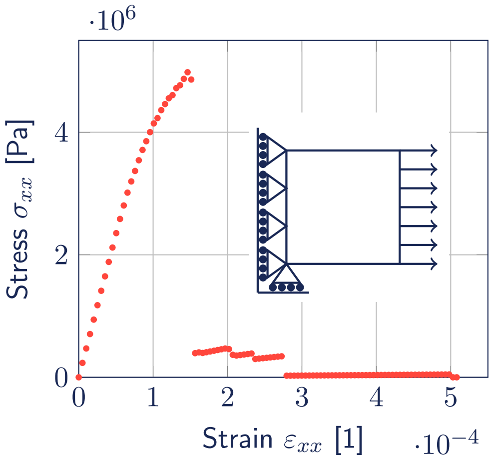

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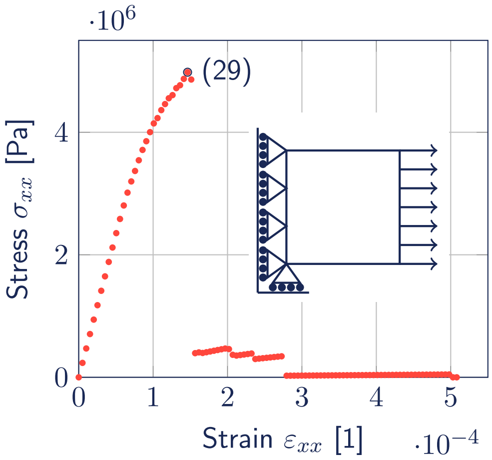
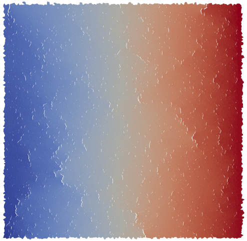


Step 29
Stress peak



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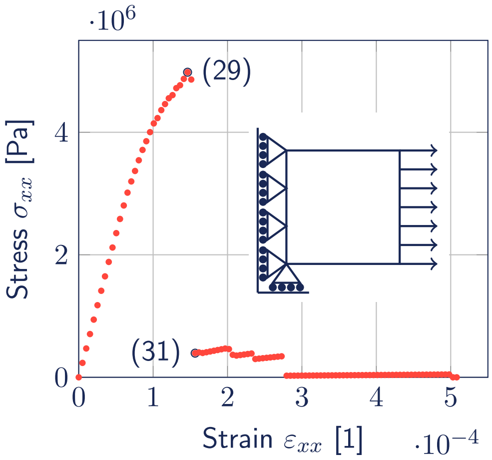

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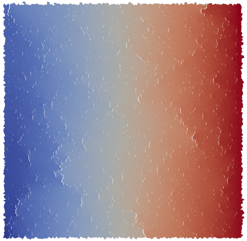
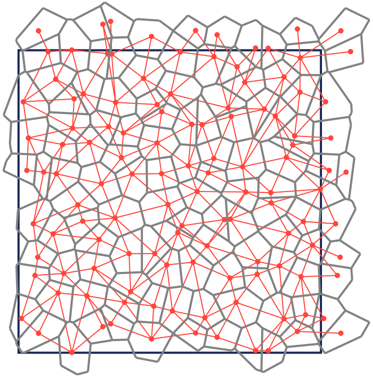


Step 31
Post peak

Measurement of the elasticity tensor – Idea

Representative Volume Element (RVE)



How to measure the effective elasticity tensor ?

Measurement of the elasticity tensor – Idea

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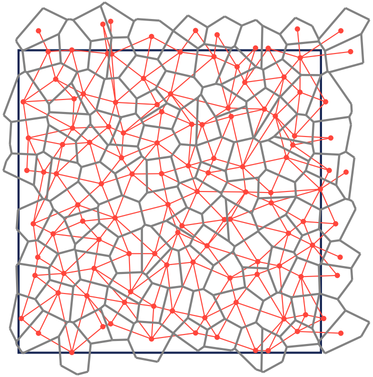


How to measure the effective elasticity tensor ?

Given,

- $\underset{\sim}{\varepsilon}^{(i)}$ – 3 linearly independant strains
- $\underset{\sim}{\sigma}^{(i)}$ – 3 associated stresses

the effective elasticity tensor is given by,

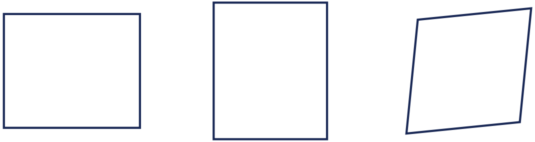
$$\underset{\approx}{E}(\mathbf{D}) = \left(\left[\underset{\sim}{\sigma}^{(1)} \middle| \underset{\sim}{\sigma}^{(2)} \middle| \underset{\sim}{\sigma}^{(3)} \right] \left[\underset{\sim}{\varepsilon}^{(1)} \middle| \underset{\sim}{\varepsilon}^{(2)} \middle| \underset{\sim}{\varepsilon}^{(3)} \right]^{-1} \right)^{S}$$

Kelvin notation: $\underset{\sim}{\sigma}^{(i)} = \left[\sigma_{xx}^{(i)}, \sigma_{yy}^{(i)}, \sqrt{2}\sigma_{xy}^{(i)} \right]^{T}$

Measurement of the elasticity tensor – Procedure

1. Apply 3 periodic elastic loadings

$$\mathbf{u}(\mathbf{x}+\Delta\mathbf{x}) = \mathbf{u}(\mathbf{x}) + \boldsymbol{\varepsilon}_{imp} \cdot \mathbf{x}$$



$$\boldsymbol{\varepsilon}_{imp}^{(i)} \propto \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad \boldsymbol{\varepsilon}_{imp}^{(i)} \propto \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad \boldsymbol{\varepsilon}_{imp}^{(i)} \propto \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

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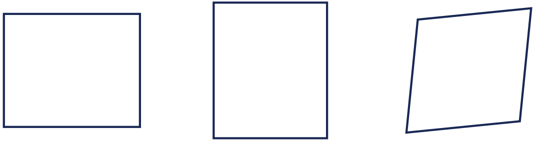

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$$\bar{\varepsilon} = \frac{1}{V} \sum_{b=0}^{N_b} \frac{\mathbf{u}_b^{(1)} + \mathbf{u}_b^{(2)}}{2} \odot \mathbf{n}_b$$

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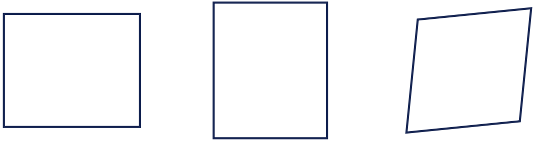

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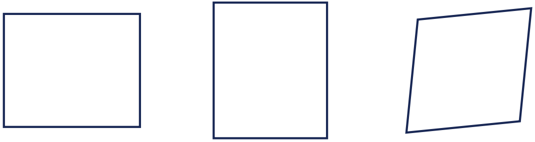

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4. Calculate elasticiy tensor

$$\underline{\underline{E}}(\mathbf{D}) = \left(\left[\underline{\sigma}^{(1)} \middle| \underline{\sigma}^{(2)} \middle| \underline{\sigma}^{(3)} \right] \left[\underline{\varepsilon}^{(1)} \middle| \underline{\varepsilon}^{(2)} \middle| \underline{\varepsilon}^{(3)} \right]^{-1} \right)^S$$

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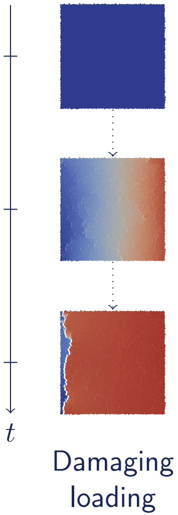

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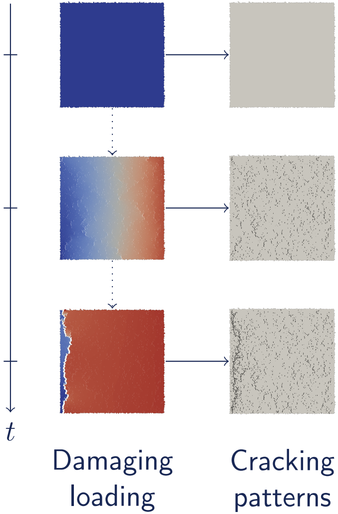

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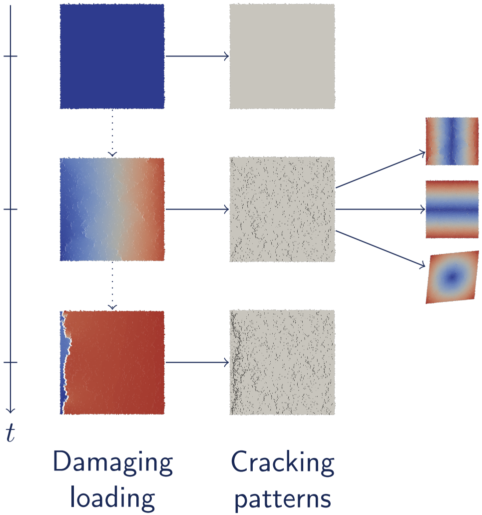

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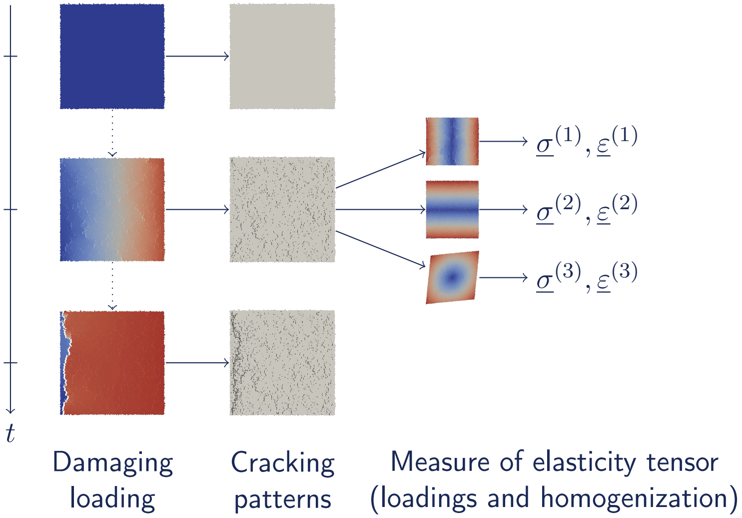

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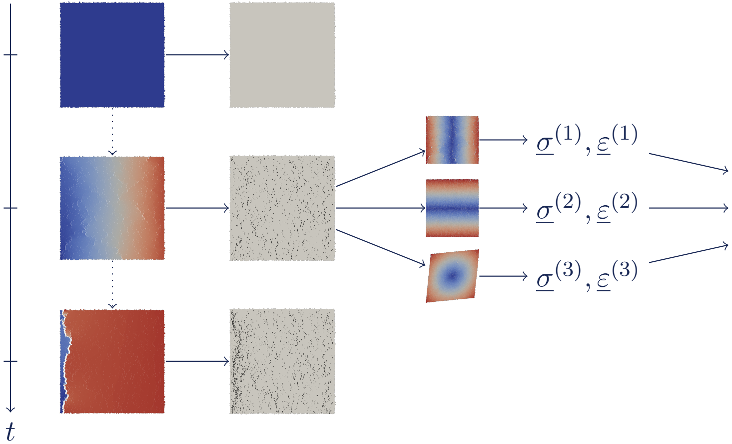

$$\begin{bmatrix} 36.68 & 8.80 & -0.20 \\ 8.80 & 48.30 & -0.02 \\ -0.20 & -0.02 & 33.68 \end{bmatrix}$$

Damaging loading | Cracking patterns | Measure of elasticity tensor (loadings and homogenization) | Elasticity tensor in Kelvin notation [MPa]

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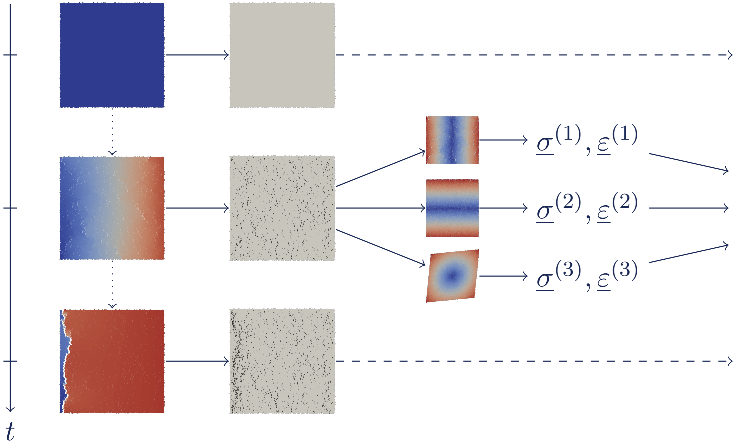


$$\begin{bmatrix} 49.74 & 10.26 & -0.11 \\ 10.26 & 49.74 & 0.11 \\ -0.11 & 0.11 & 38.20 \end{bmatrix}$$

$$\begin{bmatrix} 36.68 & 8.80 & -0.20 \\ 8.80 & 48.30 & -0.02 \\ -0.20 & -0.02 & 33.68 \end{bmatrix}$$

$$\begin{bmatrix} 28.35 & 0.37 & -0.57 \\ 0.37 & 43.65 & 0.25 \\ -0.57 & 0.25 & 10.07 \end{bmatrix}$$

Damaging loading

Cracking patterns

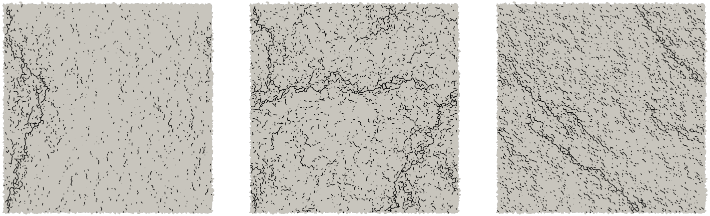
Measure of elasticity tensor (loadings and homogenization)

Elasticity tensor in Kelvin notation [MPa]

Virtual testing – Conclusion

Dataset generation

- 36 micro-structures
- 21 loadings
- 756 evolutions
- 76 356 patterns



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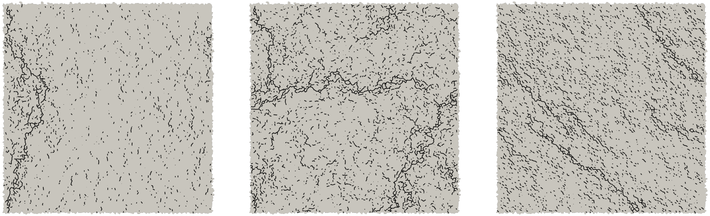


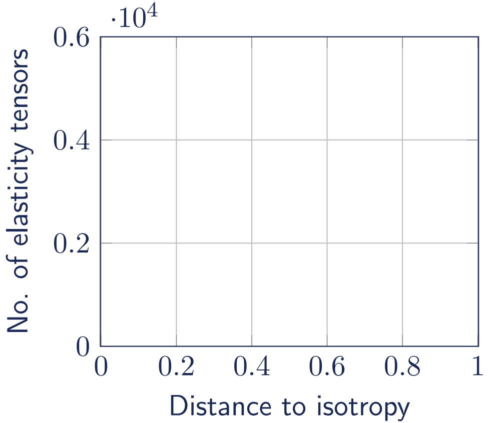
Intermediate conclusion

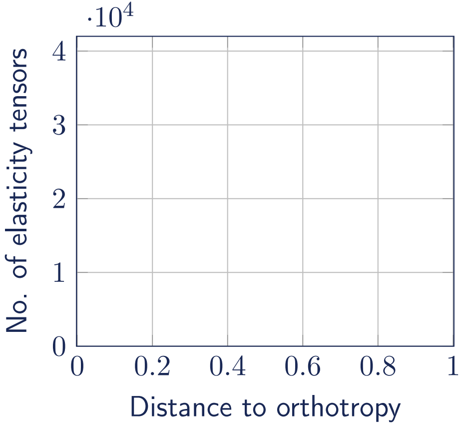
- Measurement of elasticity tensor from beam-particle model
- Generation of a dataset

Damage variable

How to represent damage ?

Distance to a symmetry stratum Σ : $$\Delta_\Sigma(\mathbf{E}) = \min_{\mathbf{E}^* \in \Sigma} \frac{\|\mathbf{E} - \mathbf{E}^*\|}{\|\mathbf{E}\|}$$





Distance to orthotropy in 2D: (Antonelli et al., 2022)

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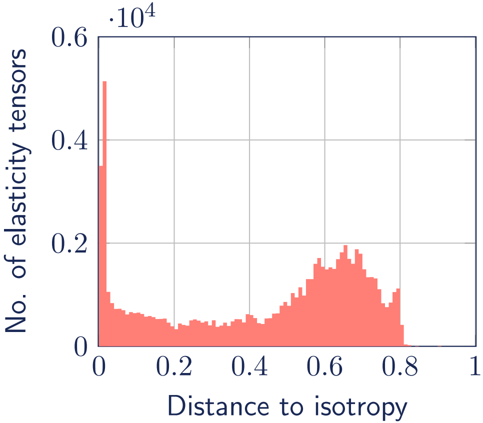

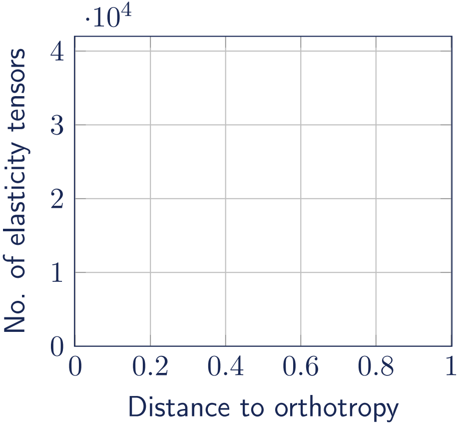

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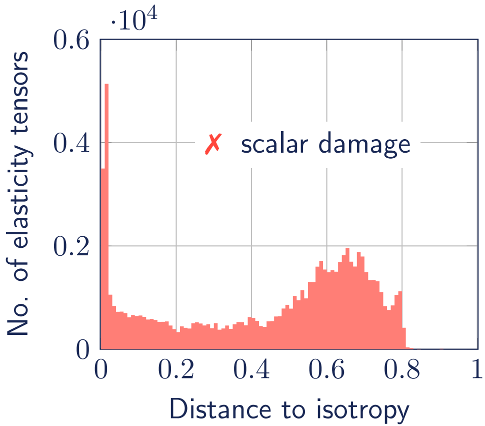

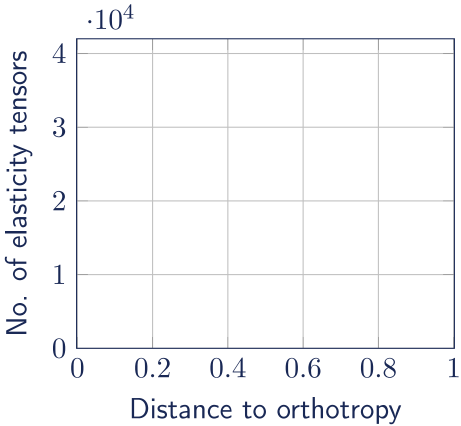

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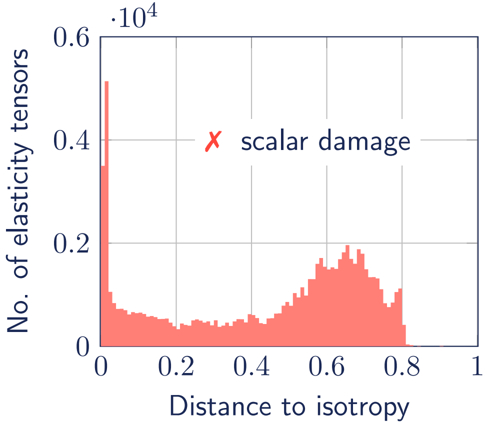

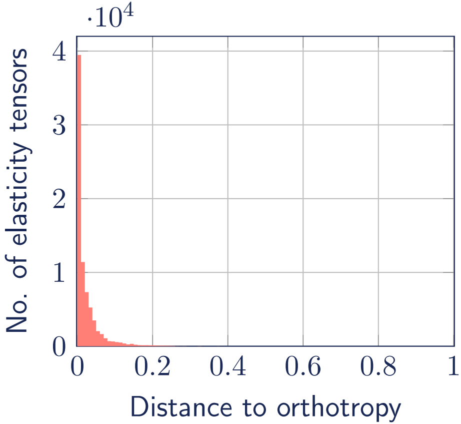

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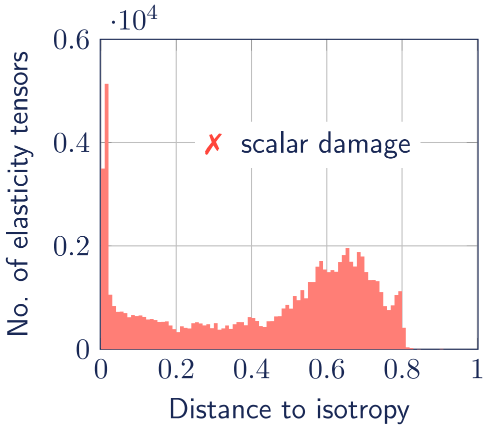

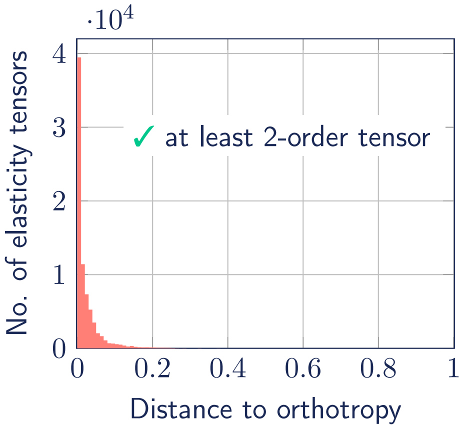

Distance to orthotropy in 2D: (Antonelli et al., 2022)

2D Harmonic decomposition (Backus, 1970), (Blinowski et al., 1996)

$$\mathbf{E} \cong (\mu, \kappa, \mathbf{d}', \mathbf{H}) \in \mathbb{H}^1(\mathbb{R}^2) \oplus \mathbb{H}^1(\mathbb{R}^2) \oplus \mathbb{H}^2(\mathbb{R}^2) \oplus \mathbb{H}^4(\mathbb{R}^2)$$

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Reconstruction $$(\mu, \kappa, \mathbf{d}', \mathbf{H}) \mapsto \mathbf{E} = \underbrace{2\mu\mathbf{J} + \kappa\mathbf{1}_2 \otimes \mathbf{1}_2}_{\mathbf{Iso}} + \underbrace{\frac{1}{2}\left(\mathbf{d}' \otimes \mathbf{1}_2 + \mathbf{1}_2 \otimes \mathbf{d}'\right)}_{\mathbf{Dil}} + \mathbf{H}$$

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Decomposition $\mathbf{E} \mapsto (\mu, \kappa, \mathbf{d}', \mathbf{H})$ where $\mathbf{d} = \operatorname{tr}_{12}\mathbf{E}$, $\mathbf{v} = \operatorname{tr}_{13}\mathbf{E}$,

$\begin{cases} \mu = \frac{1}{8}(2\operatorname{tr}\mathbf{v} - \operatorname{tr}\mathbf{d}) \\ \kappa = \frac{1}{4}\operatorname{tr}\mathbf{d} \end{cases}$ are invariants and, $\begin{cases} \mathbf{d}' = \mathbf{d} - \frac{1}{2}\operatorname{tr}\mathbf{d}\;\mathbf{1}_2 \\ \mathbf{H} = \mathbf{E} - \mathbf{Iso} - \mathbf{Dil} \end{cases}$ are covariants.

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Damage variable $\mathbf{D} = (\mathbf{d}_0 - \mathbf{d}) \cdot \mathbf{d}_0^{-1} = \mathbf{1}_2 - \dfrac{1}{2\kappa_0} \mathbf{d} \iff \mathbf{d} = 2\kappa_0 (\mathbf{1}_2 - \mathbf{D})$

2D Harmonic decomposition (Backus, 1970), (Blinowski et al., 1996)

$$\mathbf{E} \cong (\mu, \kappa, \mathbf{d}', \mathbf{H}) \in \mathbb{H}^1(\mathbb{R}^2) \oplus \mathbb{H}^1(\mathbb{R}^2) \oplus \mathbb{H}^2(\mathbb{R}^2) \oplus \mathbb{H}^4(\mathbb{R}^2)$$

Reconstruction $(\mu, \kappa, \mathbf{d}', \mathbf{H}) \mapsto \mathbf{E} = \underbrace{2\mu\mathbf{J} + \kappa \mathbf{1}_2 \otimes \mathbf{1}_2}_{\mathbf{Iso}} + \underbrace{\frac{1}{2}\left(\mathbf{d}' \otimes \mathbf{1}_2 + \mathbf{1}_2 \otimes \mathbf{d}'\right)}_{\mathbf{Dil}} + \mathbf{H}$

Decomposition $\mathbf{E} \mapsto (\mu, \kappa, \mathbf{d}', \mathbf{H})$ where $\mathbf{d} = \operatorname{tr}_{12}\mathbf{E}$, $\mathbf{v} = \operatorname{tr}_{13}\mathbf{E}$,

$\begin{cases} \mu = \frac{1}{8}(2\operatorname{tr}\mathbf{v} - \operatorname{tr}\mathbf{d}) \\ \kappa = \frac{1}{4}\operatorname{tr}\mathbf{d} \end{cases}$ are invariants and, $\begin{cases} \mathbf{d}' = \mathbf{d} - \frac{1}{2}\operatorname{tr}\mathbf{d}\,\mathbf{1}_2 \\ \mathbf{H} = \mathbf{E} - \mathbf{Iso} - \mathbf{Dil} \end{cases}$ are covariants.

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Damage variable – Conclusion

Distance to symmetry strata

$$\Delta_{\mathcal{C}}(\mathbf{E}) = \min_{\mathbf{E}^* \in \mathcal{C}} \frac{\|\mathbf{E} - \mathbf{E}^*\|}{\|\mathbf{E}\|}$$

- Indicates the type of damage variable
- At least a 2-order damage tensor

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Harmonic decomposition

$$(\mu, \kappa, \mathbf{d}', \mathbf{H}) \mapsto \mathbf{E} = 2\mu\mathbf{J} + \kappa\mathbf{1}_2 \otimes \mathbf{1}_2 + \frac{1}{2}\left(\mathbf{d}' \otimes \mathbf{1}_2 + \mathbf{1}_2 \otimes \mathbf{d}'\right) + \mathbf{H}$$

- Guide the definition of the damage variable $\mathbf{D} = \mathbf{1}_2 - \frac{1}{2\kappa_0}\mathbf{d}$
- $\kappa(\mathbf{D})$ and $\mathbf{d}'(\mathbf{D})$ are exact
- $\mu(\mathbf{D})$ and $\mathbf{H}(\mathbf{D})$ need to be modelled

State model

Shear modulus $\mu = \frac{1}{8}\left(2\,\mathrm{tr}\,\mathbf{v} - \mathrm{tr}\,\mathbf{d}\right)$ model as a function of damage

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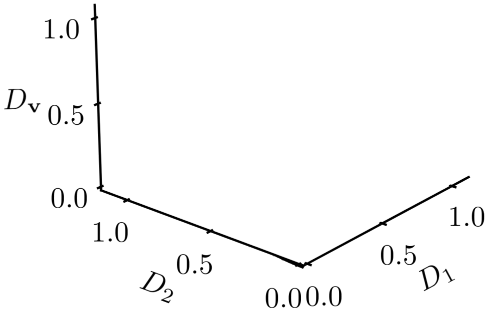
$$\iff \mathrm{tr}\,\mathbf{v} = \mathrm{tr}\,\mathbf{v}_0(1 - D_{\mathbf{v}}).$$



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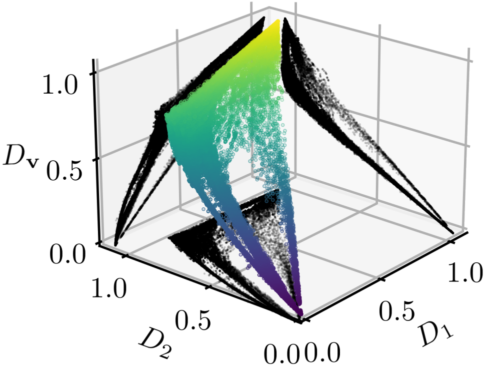

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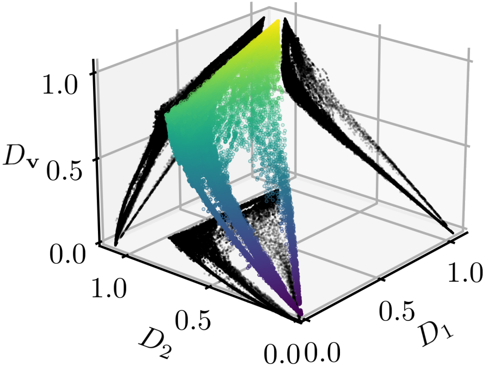

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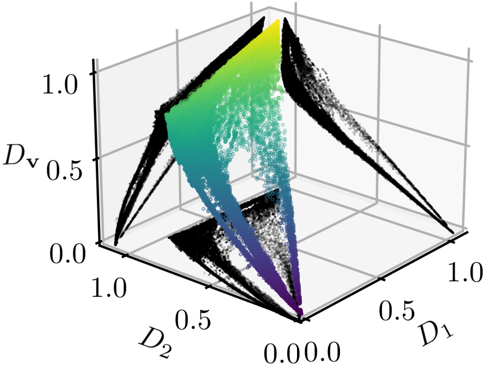

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$$D_{\mathbf{v}}^{\mathrm{m},2} = c_1 I_1(\mathbf{D}) + c_2 I_2(\mathbf{D}) \quad \text{where} \begin{cases} D_{\mathbf{v}}(\mathbf{0}) = 0 & \Longrightarrow \text{OK} \\ D_{\mathbf{v}}(\mathbf{1}_2) = 1 & \Longrightarrow c_2 = \frac{1}{2} - c_1 \\ \left.\frac{\partial D_{\mathbf{v}}}{\partial \mathbf{D}}\right|_{\mathbf{D}=\mathbf{0}} = \frac{\kappa_0}{2\mu_0 + \kappa_0}\mathbf{1}_2 & \Longrightarrow c_1 = \frac{\kappa_0}{2\mu_0 + \kappa_0} \end{cases}$$

Analysis of the harmonic part

Harmonic square reconstruction (Oliver-Leblond et al., 2021)

For an orthotropic elasticity tensor $\mathbf{E}_O$, the harmonic part is

$$\mathbf{H}(\mathbf{E}_O) = \frac{2K_3}{I_2^2}\mathbf{d}' * \mathbf{d}'$$

where:

- $K_3 = \mathbf{d} : \mathbf{H} : \mathbf{d}$,
- $I_2 = \mathbf{d}' : \mathbf{d}'$,
- for a 2D 2-order harmonic tensor $\mathbf{d}'$,
  $$\mathbf{d}' * \mathbf{d}' = \mathbf{d}' \otimes \mathbf{d}' - \frac{1}{2}(\mathbf{d}' : \mathbf{d}')\,\mathbf{J}.$$

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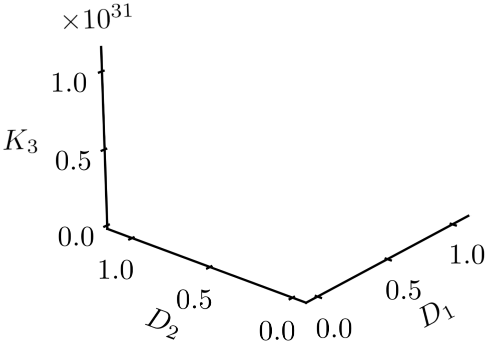
Can K_3 be modelled using damage ?



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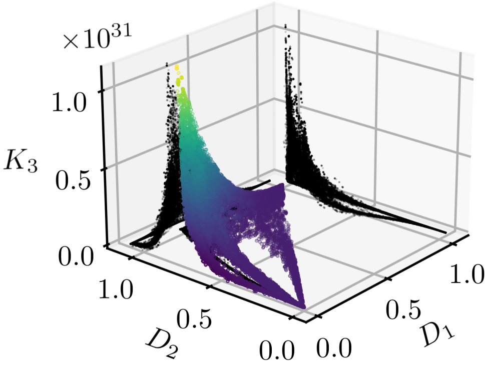


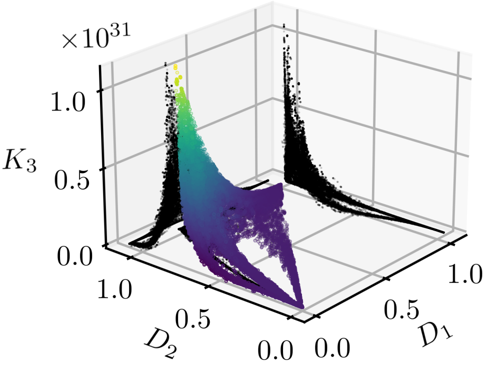
Harmonic part H as a function of damage

Solution 1

Solution 0

Harmonic part H as a function of damage

Solution 1



Work in progress: Model $K_3(\mathbf{D})$

Solution 0

Harmonic part $\mathbb{H}$ as a function of damage

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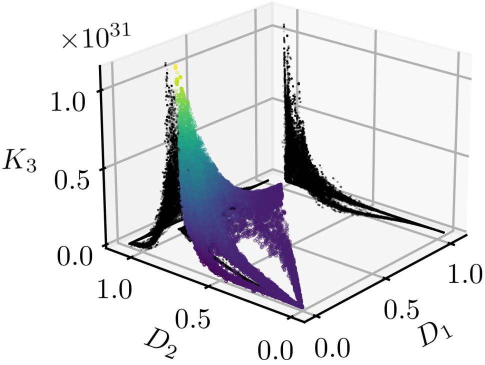

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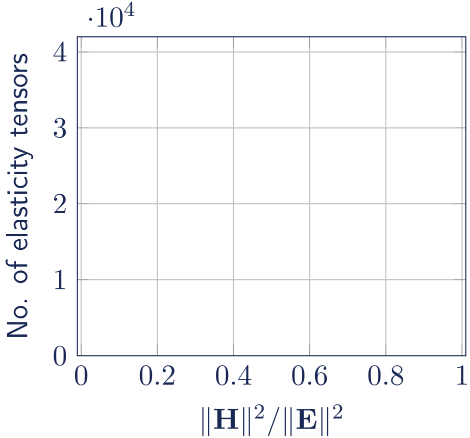

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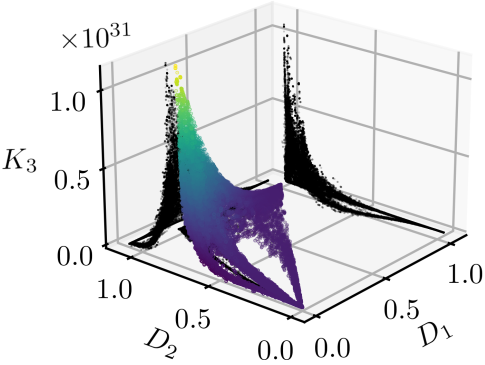

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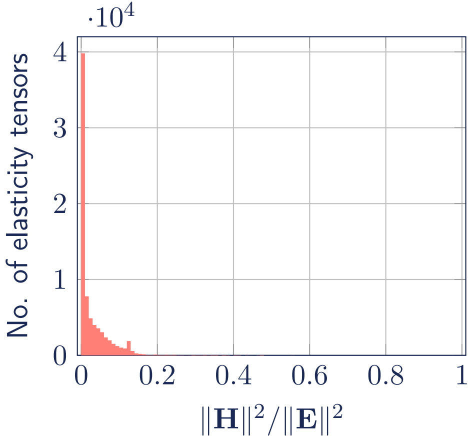

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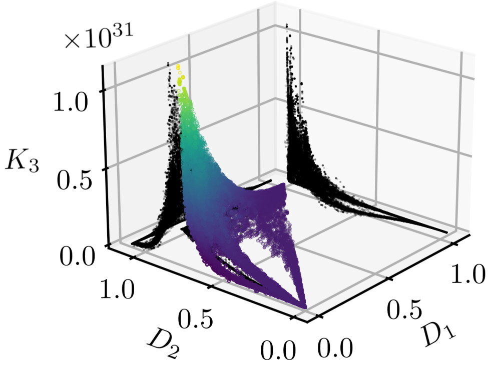

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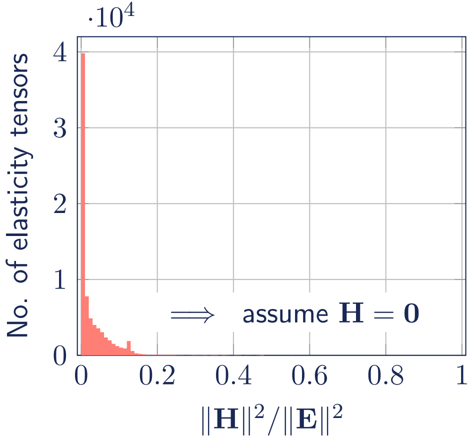


Summary of model

If $\mathbf{E}_0$ and $\mathbf{D}$ are known, the elasticity tensor can be modelled as

$$\mathbf{E}(\mathbf{D}) = 2\mu(\mathbf{D})\mathbf{J} + \kappa(\mathbf{D})\mathbf{1}_2 \otimes \mathbf{1}_2 + \frac{1}{2}\left(\mathbf{d}'(\mathbf{D}) \otimes \mathbf{1}_2 + \mathbf{1}_2 \otimes \mathbf{d}'(\mathbf{D})\right) + \mathbf{H}(\mathbf{D})$$

where,

$$\mathbf{d}(\mathbf{D}) = 2\kappa_0(\mathbf{1}_2 - \mathbf{D})$$

$$\kappa(\mathbf{D}) = \frac{1}{4}\operatorname{tr}\mathbf{d}$$

$$\mathbf{d}'(\mathbf{D}) = \mathbf{d} - \frac{1}{2}\operatorname{tr}\mathbf{d}\ \mathbf{1}_2$$

are exact.

$$D_{\mathbf{v}}^{\mathrm{m},2}(\mathbf{D}) = c_1 I_1(\mathbf{D}) + \left(\frac{1}{2} - c_1\right) I_2(\mathbf{D})$$

$$\mu^{\mathrm{m}}(\mathbf{D}) = \frac{1}{8}\left(2\operatorname{tr}\mathbf{v}_0(1 - D_{\mathbf{v}}^{\mathrm{m}}) - \operatorname{tr}\mathbf{d}\right)$$

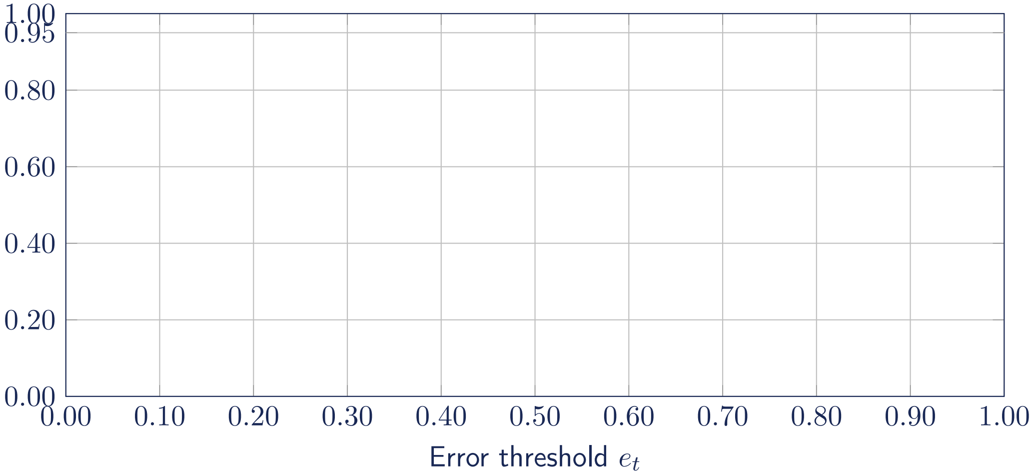
$$\mathbf{H}^{\mathrm{m}}(\mathbf{D}) = \mathbf{0}$$

are approximate.

The parameter $c_1 = \frac{\kappa_0}{2\mu_0+\kappa_0}$ and the invariants of $\mathbf{D}$ are $I_n = \operatorname{tr}(\mathbf{D}^n)$.

Model error estimation

Proportion of $\mathbf{E}$ s.t. $\frac{\|\mathbf{E}-\mathbf{E}^{\mathrm{m}}\|}{\|\mathbf{E}_0\|} \leq e_t$



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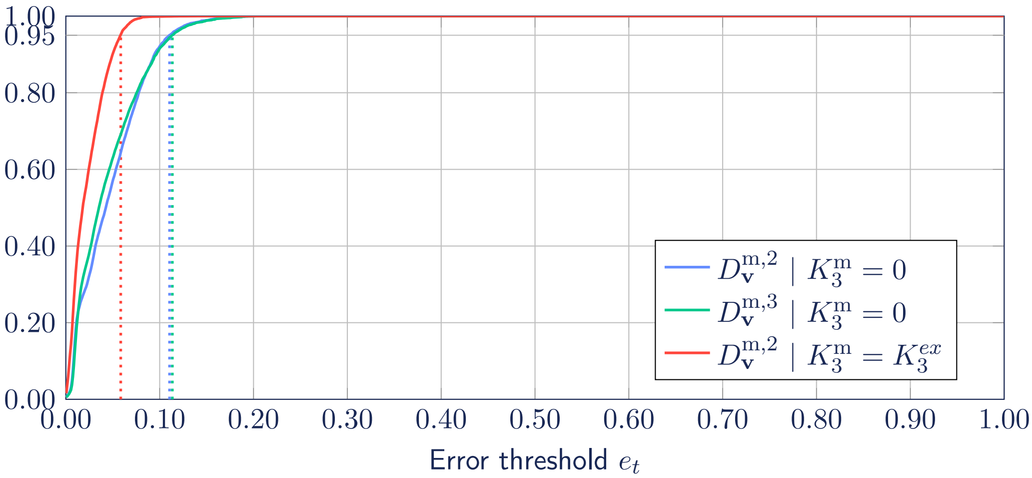

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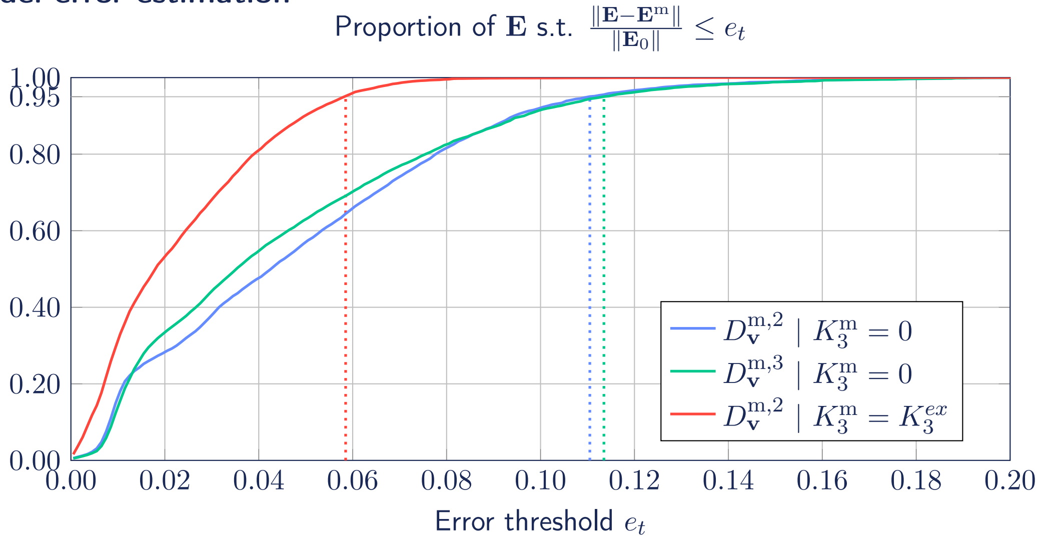

Conclusion

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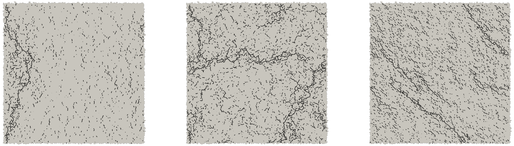
Objectives of the presentation

1. Gather data on the behavior of quasi-brittle material
   - Virtual testing



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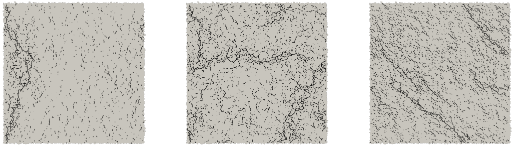

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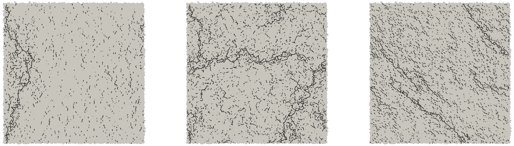

$$\mathbf{D} = \mathbf{1}_2 - \frac{1}{2\kappa_0}\mathbf{d}$$

2. Quantify the micro-cracking
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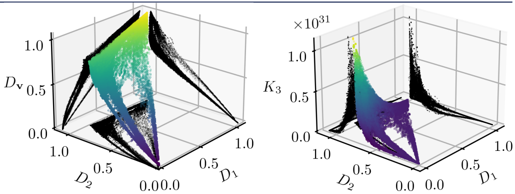
3. Model the impact of micro-cracking on the relation between $\boldsymbol{\varepsilon}$ and $\boldsymbol{\sigma}$
   - Exact models of $\kappa(\mathbf{D})$, $\mathbf{d}'(\mathbf{D})$
   - Modelling of $\mu(\mathbf{D})$, $\mathbf{H}(\mathbf{D})$



Perspectives

Objective of the project

Formulating an anisotropic damage model for quasi-brittle material

Formulation of a damage model

$$\mathcal{V} = \{\varepsilon, \mathbf{D}, ...\} \qquad \text{(Set of variables)}$$

$$\rho\psi = \frac{1}{2}\varepsilon : \mathbf{E}(\mathbf{D}) : \varepsilon \qquad \text{(State potential)}$$

$$\dot{\mathbf{D}} = ... \qquad \text{(Damage evolution)}$$

Perspectives

Constraints

- $\mathbf{E}(\mathbf{D})$ is positive definite
- Positive dissipation

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- Formulate a model of K_3 as a function of $\mathbf{D}$
- Formulate an evolution model for $\mathbf{D}$
- Study nonlocal damage via beam-particle model

Merci pour votre attention !

Réunion thématique du GDR-GDM, ENS Paris-Saclay, 23/11/2022.

Flavien Loiseau – flavien.loiseau@ens-paris-saclay.fr
Encadré par: Rodrigue Desmorat, Cécile Oliver-Leblond

Université Paris-Saclay, CentraleSupélec, ENS Paris-Saclay, CNRS, LMPS - Laboratoire de Mécanique Paris-Saclay, 91190, Gif-sur-Yvette, France.

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